

UPPSC-AE

2025

Uttar Pradesh Public Service Commission

Combined State Engineering Services Examination
Assistant Engineer

Electrical Engineering

Electromagnetic Theory

Well Illustrated **Theory** *with*
Solved Examples and Practice Questions



Note: This book contains copyright subject matter to MADE EASY Publications, New Delhi. No part of this book may be reproduced, stored in a retrieval system or transmitted in any form or by any means. Violators are liable to be legally prosecuted.

Electromagnetic Theory

Contents

UNIT	TOPIC	PAGE NO.
1.	Vector Analysis	3 - 24
2.	Electrostatics	25 - 58
3.	Magnetostatics	59 - 73
4.	Time-Varying Electromagnetic fields	74 - 82
5.	Electromagnetic Waves	83 - 104
6.	Ground and Sky wave propagation	105 - 116
7.	Satellite Communication	117 - 132



Vector Analysis

1.1 Introduction

The quantities of interest appearing in the study of EM theory can almost be classified as either a scalar or a vector. Quantities that can be described by a magnitude alone are called scalars. Distance, temperature, mass etc. are examples of scalar quantities. Quantities, that require both a magnitude and a direction to fully characterize them are vectors. Vector quantities include velocity, force, acceleration etc are examples of vector quantities.

In electromagnetics, we frequently use the concept of a **field**. A field is a function that assigns a particular physical quantity to every point in a region. In general, a field varies with both position and time. There are scalar fields and vector fields. Temperature distribution in a room and electric potential are examples of scalar fields. Electric field and magnetic flux density are examples of vector fields.

NOTE: Vectors are denoted by an arrow over a letter (\vec{A}) and scalars are denoted by simple letter (A).

1.1.1 Unit Vector

- A unit vector \hat{a}_A along \vec{A} is defined as a vector whose magnitude is unity (*i.e.*, 1) and its direction is along \vec{A} , that is

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

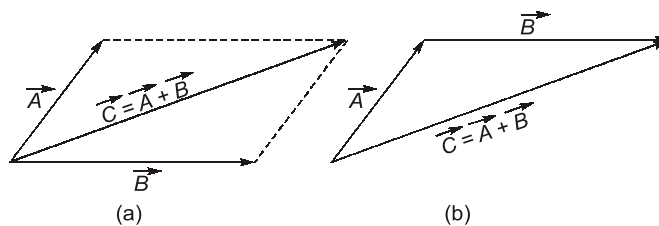
Thus we can write \vec{A} as $\vec{A} = A \hat{a}_A = |\vec{A}| \hat{a}_A$

REMEMBER: Any vector can be written as product of its magnitude and its unit vector.

1.1.2 Vector Addition and Subtraction

Two vectors \vec{A} and \vec{B} can be added together to give another vector \vec{C} ; that is,

$$\vec{C} = \vec{A} + \vec{B}$$



Vector addition (a) parallelogram rule, (b) head-to-tail rule.

- Vector subtraction is similarly carried out as

$$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

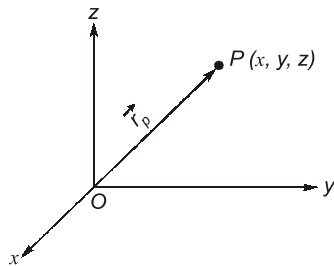
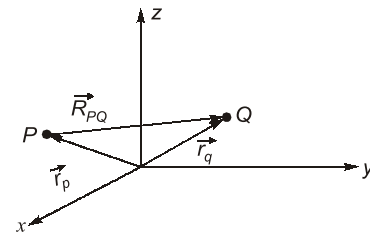
**NOTE**

- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ (Commutative law)
- $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$ (Associative law)
- $k(\vec{A} + \vec{B}) = k\vec{A} + k\vec{B}$ (Distributive law)
- $\frac{\vec{A} + \vec{B}}{k} = \frac{1}{k}\vec{A} + \frac{1}{k}\vec{B}$

1.1.3 Position and Distance Vectors:

- A point P in Cartesian coordinates may be represented by (x, y, z) .
- The position vector \vec{r}_p (or radius vector) of point P is defined as the directed distance from origin O to P .

$$\vec{r}_p = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

Illustration of position vector $\vec{r}_p = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ Vector distance \vec{R}_{PQ}

- The distance vector is the displacement from one point to another.
- Consider point P with position vector \vec{r}_p and point Q with position vector \vec{r}_q . The displacement from P to Q is written as

$$\vec{R}_{PQ} = \vec{r}_q - \vec{r}_p$$



will be

Example - 1.1 Point P and Q are located at $(0, 2, 4)$ and $(-3, 1, 5)$. The position vector P

- (a) $-3\hat{a}_x + \hat{a}_y + 5\hat{a}_z$ (b) $-3\hat{a}_x + 5\hat{a}_z$
 (c) $2\hat{a}_y + 4\hat{a}_z$ (d) $2\hat{a}_x - 4\hat{a}_z$

Solution: (c)

$$\vec{r}_p = 0\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z = 2\hat{a}_y + 4\hat{a}_z$$

from P to Q will be**Example - 1.2** Point P and Q are located at $(0, 2, 4)$ and $(-3, 1, 5)$. The distance vector

- (a) $-3\hat{a}_x - \hat{a}_y + \hat{a}_z$ (b) $-3\hat{a}_x - \hat{a}_y - \hat{a}_z$
 (c) $3\hat{a}_x + \hat{a}_y + \hat{a}_z$ (d) $3\hat{a}_x - \hat{a}_y + \hat{a}_z$

Solution: (a)

$$\begin{aligned}\vec{R}_{PQ} &= \vec{r}_Q - \vec{r}_P = (-3, 1, 5) - (0, 2, 4) = (-3, -1, 1) \\ &= -3\hat{a}_x - \hat{a}_y + \hat{a}_z\end{aligned}$$

1.1.4 Vector Multiplication

- When two vectors are multiplied, the result is either a scalar or a vector depending on how they are multiplied. Thus there are two types of vector multiplication.

- Scalar (or dot) product : $\vec{A} \cdot \vec{B}$
- Vector (or cross) product : $\vec{A} \times \vec{B}$
Multiplication of three vectors \vec{A} , \vec{B} , \vec{C} can result in either
- Scalar triple product : $\vec{A} \cdot (\vec{B} \times \vec{C})$
- Vector triple product : $\vec{A} \times (\vec{B} \times \vec{C})$

Dot Product:

- The dot product, or the scalar product of two vectors \vec{A} and \vec{B} , written as $\vec{A} \cdot \vec{B}$ is defined geometrically as the product of the magnitudes of \vec{A} and \vec{B} and the cosine of the angle between them.

$$\vec{A} \cdot \vec{B} = A B \cos \theta_{AB}$$

Where θ_{AB} is the smaller angle between \vec{A} and \vec{B} . The result of $\vec{A} \cdot \vec{B}$ is called either the scalar product because it is scalar, or the dot product due to the dot sign.

If $\vec{A} = (A_x, A_y, A_z)$

and $\vec{B} = (B_x, B_y, B_z)$

then $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

NOTE: Two vectors \vec{A} and \vec{B} are said to be orthogonal (or perpendicular) with each other if $\vec{A} \cdot \vec{B} = 0$.

- The dot product obeys the following laws:

Commutative Law

Expression: $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Distributive Law

Expression: $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
 $\vec{A} \cdot \vec{A} = |\vec{A}|^2 = A^2$

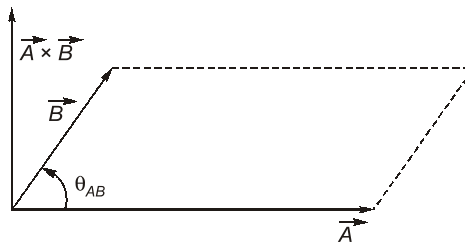


NOTE

- $\hat{a}_x \cdot \hat{a}_y = \hat{a}_y \cdot \hat{a}_z = \hat{a}_z \cdot \hat{a}_x = 0$
- $\hat{a}_x \cdot \hat{a}_x = \hat{a}_y \cdot \hat{a}_y = \hat{a}_z \cdot \hat{a}_z = 1$

Cross Product:

- The cross product of two vectors \vec{A} and \vec{B} , written as $\vec{A} \times \vec{B}$, is a vector quantity whose magnitude is the area of the parallelepiped formed by \vec{A} and \vec{B} and is in the direction of advance of the right-handed screw as \vec{A} is turned into \vec{B} .



The cross product of \vec{A} and \vec{B} is a vector with magnitude equal to the area of parallelogram and the direction as indicated

$$\vec{A} \times \vec{B} = AB \sin \theta_{AB} \hat{a}_n$$

where \hat{a}_n is a unit vector normal to the plane containing \vec{A} and \vec{B} .

- The vector multiplication of equation is called **cross product** due to the cross sign. It is also called **vector product** because the result is a vector.

If $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$ then :

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{a}_x + (A_z B_x - A_x B_z) \hat{a}_y + (A_x B_y - A_y B_x) \hat{a}_z$$

Also $\vec{A} \times \vec{B} = 0$, then $\sin \theta_{AB} = 0^\circ$ or 180° ; this shows that \vec{A} and \vec{B} are parallel or antiparallel to each other.

- Above result is obtained by 'crossing' terms in cyclic permutation, hence the name cross product. Note that the cross product has the following properties

1. It is not commutative:

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

2. It is not associative:

$$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$

3. It is distributive:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$



NOTE

For a vector $\vec{A} = \hat{a}_x + \hat{a}_y + \hat{a}_z$

- $\vec{A} \times \vec{A} = 0$
- $\hat{a}_x \times \hat{a}_y = \hat{a}_z, \hat{a}_y \times \hat{a}_z = \hat{a}_x, \hat{a}_z \times \hat{a}_x = \hat{a}_y$

Scalar Triple Product:

- Given three vectors \vec{A} , \vec{B} , and \vec{C} , we define scalar triple product as,

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

- If $\vec{A} = (A_x, A_y, A_z)$, $\vec{B} = (B_x, B_y, B_z)$ and $\vec{C} = (C_x, C_y, C_z)$, then $\vec{A} \cdot (\vec{B} \times \vec{C})$ is the volume of a parallelepiped having \vec{A} , \vec{B} , and \vec{C} as edges and is easily obtained by finding the determinant of the 3×3 matrix formed by \vec{A} , \vec{B} , and \vec{C} ; that is

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

- Since the result of this vector multiplication is scalar these two equations are called the scalar triple product.

Vector Triple Product:

- For vectors \vec{A} , \vec{B} , and \vec{C} , we define the vector triple product as

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

This is obtained using the “bac-cab” rule.



Example - 1.3 Three field quantities are given by $\vec{P} = 2\hat{a}_x - \hat{a}_z$ and $\vec{Q} = 2\hat{a}_x - \hat{a}_y + 2\hat{a}_z$,

$\vec{R} = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$. The value of $(\vec{P} + \vec{Q}) \times (\vec{P} - \vec{Q})$ is

- | | |
|---|--|
| (a) $2\hat{a}_x - 12\hat{a}_y + 4\hat{a}_z$ | (b) $2\hat{a}_x + 12\hat{a}_y + 4\hat{a}_z$ |
| (c) $2\hat{a}_x - 12\hat{a}_y - 4\hat{a}_z$ | (d) $-2\hat{a}_x - 12\hat{a}_y - 4\hat{a}_z$ |

Solution: (b)

$$\begin{aligned} (\vec{P} + \vec{Q}) \times (\vec{P} - \vec{Q}) &= 2(\vec{Q} \times \vec{P}) \\ &= 2 \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} \\ &= 2(1 - 0)\hat{a}_x + 2(4 + 2)\hat{a}_y + 2(0 + 2)\hat{a}_z \\ &= 2\hat{a}_x + 12\hat{a}_y + 4\hat{a}_z \end{aligned}$$



Example - 1.4 Three field quantities are given by $\vec{P} = 2\hat{a}_x - \hat{a}_z$ and $\vec{Q} = 2\hat{a}_x - \hat{a}_y + 2\hat{a}_z$,

$\vec{R} = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$. The value of $\vec{Q} \cdot (\vec{R} \times \vec{P})$ is

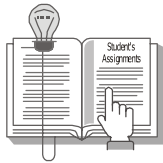
- | | |
|--------|--------|
| (a) 10 | (b) 18 |
| (c) 2 | (d) 14 |

Solution: (d)

$$\begin{aligned} \vec{Q} \cdot (\vec{R} \times \vec{P}) &= (2, -1, 2) \cdot \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix} \\ &= (2, -1, 2) \cdot (3, 4, 6) \\ &= 6 - 4 + 12 = 14 \end{aligned}$$

Alternatively:

$$\vec{Q} \cdot (\vec{R} \times \vec{P}) = \begin{vmatrix} 2 & -1 & 2 \\ 2 & -3 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 14$$



Student's Assignment

Q.1 A point is represented in Cartesian coordinates as $P(3, 4, 5)$, the radial component ρ in cylindrical coordinates will be _____ r in spherical coordinates.

- (a) less than (b) greater than
(c) equal to (d) unrelated to

Q.2 Consider a vector $\vec{E} = z\hat{a}_x + (x + y)\hat{a}_y$, the z component of the vector in cylindrical coordinates will be

- (a) z
(b) $z \cos \phi + (x + y) \sin \phi$
(c) $-z \sin \phi + (x + y) \cos \phi$
(d) zero

Q.3 Let a point in spherical and cylindrical coordinates are (r, θ, ϕ) and (ρ, ϕ, z) . The radial component r in spherical coordinates is related to components in cylindrical coordinates as

- (a) ρ (b) $\rho \cos \phi$
(c) $z \tan^{-1} \phi$ (d) $(\rho^2 + z^2)^{1/2}$

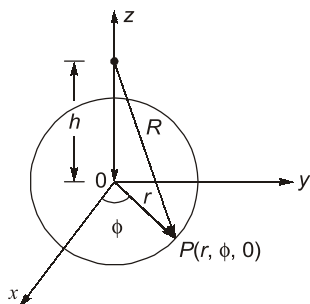
Q.4 Given the vector

$$\vec{A} = (\cos x)(\sin y)\hat{a}_x + (\sin x)(\cos y)\hat{a}_y,$$

where \hat{a}_x, \hat{a}_y denote unit vectors along x, y directions, respectively. The magnitude of curl of \vec{A} is

- (a) 0 (b) 1
(c) -1 (d) 2

Q.5 The unit vector \vec{a}_R which points from $z = h$ on the z -axis towards $(r, \phi, 0)$ in cylindrical co-ordinates as shown below is given by



(a) $\frac{h\vec{a}_r - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (b) $\frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$

(c) $\frac{h\vec{a}_\phi - r\vec{a}_z}{\sqrt{r^2 + h^2}}$ (d) $\frac{r\vec{a}_z - h\vec{a}_\phi}{\sqrt{r^2 + h^2}}$

Q.6 The vector field given by

$$\vec{A} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z \text{ is}$$

- (a) rotational and solenoidal
(b) rotational but not solenoidal
(c) irrotational and solenoidal
(d) irrotational but not solenoidal

Q.7 If $\vec{A} = \frac{\vec{a}_x}{\sqrt{x^2 + y^2}}$, then the value of $\nabla \cdot \vec{A}$ at

$(2, 2, 0)$ will be

- (a) -0.0884 (b) 0.0264
(c) -0.0356 (d) 0.0542

Q.8 If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then the value of

$$\vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k}) \text{ is}$$

- (a) \vec{r} (b) $2\vec{r}$
(c) $3\vec{r}$ (d) $6\vec{r}$

Q.9 What is the value of constant b so that the vector

$$\vec{V} = (x + 3y)\vec{i} + (y - 2x)\vec{j} + (x + bz)\vec{k}$$

is solenoidal?

- (a) 2 (b) -1
(c) 3 (d) -2

Q.10 If $\vec{r} = x\vec{i}_x + y\vec{i}_y + z\vec{i}_z$, then which of the following relation will hold true?

- (a) $\nabla \vec{r} = 3$
(b) $\nabla \times \vec{r} = 0$
(c) Both (a) and (b)
(d) Neither (a) nor (b)

Q.11 If $\vec{c} = \vec{a} \times \vec{b}$ and $\vec{b} = \vec{a} \times \vec{c}$, then

- (a) $\vec{b} = 0$ and $\vec{c} = 0$ (b) Only $\vec{b} = 0$
(c) Only $\vec{c} = 0$ (d) $\vec{b} \neq 0$ and $\vec{c} \neq 0$

Q.12 If S is any closed surface enclosing a volume V and $\vec{A} = ax\vec{i} + by\vec{j} + cz\vec{k}$, then the value of

$\iint_S \vec{A} \cdot \hat{n} d\vec{s}$ (\hat{n} is a unit vector) will be equal to

- (a) $\frac{1}{3}(a+b+c)V$ (b) $(a-b-c)V$
(c) $\frac{1}{2}(a+b+c)V$ (d) $(a+b+c)V$

ANSWER KEY

STUDENT'S ASSIGNMENT

1. (a) 2. (d) 3. (d) 4. (c) 5. (b)
6. (c) 7. (a) 8. (b) 9. (d) 10. (c)
11. (a) 12. (d)

HINTS & SOLUTIONS

STUDENT'S ASSIGNMENT

5. (b)

Let the unit vector be given by \vec{a}_R .

Now, $\vec{R} = \text{Difference of two vectors}$
 $= r\vec{a}_r - h\vec{a}_z$

\therefore Unit vector, $\vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\vec{a}_r - h\vec{a}_z}{\sqrt{r^2 + h^2}}$

6. (c)

The vector field \vec{A} will be irrotational, if $\nabla \times \vec{A} = 0$.

$$\begin{aligned} \text{Now, } \nabla \times \vec{A} &= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \\ &= \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz) \right] \vec{a}_x + \left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz) \right] \vec{a}_y \\ &\quad + \left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz) \right] \vec{a}_z \end{aligned}$$

$$= [x - x] \vec{a}_x + [y - y] \vec{a}_y + [z - z] \vec{a}_z = 0$$

Hence, \vec{A} is irrotational.

The vector field \vec{A} will be solenoidal, if $\nabla \cdot \vec{A} = 0$

Here,

$$\begin{aligned} \nabla \cdot \vec{A} &= \left(\vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z} \right) \cdot (yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z) \\ &= \vec{a}_x \cdot \vec{a}_x \frac{\partial}{\partial x}(yz) + \vec{a}_y \cdot \vec{a}_y \frac{\partial}{\partial y}(xz) + \vec{a}_z \cdot \vec{a}_z \frac{\partial}{\partial z}(xy) \\ &= 0 + 0 + 0 = 0 \end{aligned}$$

7. (a)

$$\text{Given, } \vec{A} = \frac{1}{\sqrt{x^2 + y^2}} \vec{a}_x$$

$$\begin{aligned} \therefore \nabla \cdot \vec{A} &= \frac{\partial}{\partial x}(A_x) + \frac{\partial}{\partial y}(A_y) + \frac{\partial}{\partial z}(A_z) \\ &= \frac{\partial}{\partial x} \left(\frac{1}{\sqrt{x^2 + y^2}} \right) + 0 + 0 \\ &= \frac{\partial}{\partial x} (x^2 + y^2)^{-1/2} \\ &= -\frac{1}{2} (x^2 + y^2)^{-3/2} \cdot 2x \\ &= \nabla \cdot \vec{A} = -\frac{x}{\sqrt{(x^2 + y^2)(x^2 + y^2)}} \end{aligned}$$

$$\begin{aligned} \text{Now, } (\nabla \cdot \vec{A})_{2,2,0} &= -\frac{2}{\sqrt{(2^2 + 2^2) \cdot (2^2 + 2^2)}} \\ &= -\frac{2}{\sqrt{8 \cdot 8}} = -0.0884 \end{aligned}$$

8. (b)

$$\text{Given, } \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\begin{aligned} \therefore \vec{r} \times \vec{i} &= (x\vec{i} + y\vec{j} + z\vec{k}) \times \vec{i} \\ &= -y\vec{k} + z\vec{j} \end{aligned}$$

$$\begin{aligned} \text{Also, } \vec{i} \times (\vec{r} \times \vec{i}) &= \vec{i} \times (-y\vec{k} + z\vec{j}) \\ &= \vec{j}y + z\vec{k} \end{aligned}$$

$$\text{Similarly, } \vec{j} \times (\vec{r} \times \vec{j}) = \vec{i}x + \vec{k}z$$

$$\text{and } \vec{k} \times (\vec{r} \times \vec{k}) = \vec{i}x + \vec{j}y$$

$$\begin{aligned} \text{Thus, } \vec{i} \times (\vec{r} \times \vec{i}) + \vec{j} \times (\vec{r} \times \vec{j}) + \vec{k} \times (\vec{r} \times \vec{k}) \\ = 2(x\vec{i} + y\vec{j} + z\vec{k}) = 2\vec{r} \end{aligned}$$